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ARTICLE

Enhancing portfolio performance: incorporating parameter uncertainties in zero-beta strategies

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Abstract

Purpose – This study examines a zero-beta portfolio strategy that accounts for the uncertainties in expected returns and betas, with the goal of improving investment performance by incorporating parameter uncertainty into the optimization process.

Theoretical framework – The research is grounded in modern portfolio theory and robust optimization, drawing on the multifactor asset pricing model of Chen, Roll, and Ross (1986). It leverages the Kalman filter to estimate dynamic betas and their uncertainties, and incorporates analysts' forecasts to assess expected returns and their associated uncertainties.

Design/methodology/approach – The study constructs two types of zero-beta portfolios: a long-short stochastic portfolio that maximizes the ratio of expected return to parameter uncertainty, and a long-short normal portfolio that focuses solely on maximizing expected return. Portfolio performance is evaluated using data from 2015 to 2022.

Findings – The results indicate that the long-short stochastic portfolios outperform the normal portfolios on several performance metrics. Specifically, they exhibit higher realized returns, lower drawdowns, and a superior realized Sharpe ratio. In addition, the stochastic approach yields more accurate predictions with a significantly lower root mean square error.

Practical & social implications of research – The findings provide insights for investors, fund managers, and practitioners seeking to improve portfolio stability and performance under uncertainty. However, reliance on analysts' estimates should be approached with caution, as deviations from expected values can still occur.

Originality/value – This study contributes to the existing literature by empirically validating the benefits of incorporating parameter uncertainty into portfolio optimization.

Keywords: Stochastic zero-beta portfolio optimization, market neutral, Kalman filter, multifactor asset pricing model.

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1 Introduction

In a situation where there are no restrictions on taking advantage of arbitrage opportunities, consider two well-diversified portfolios with identical point estimates for betas but different expected returns. In this scenario, an investor can potentially make a risk-free profit by selling (shorting) the portfolio with the lower expected return and buying the portfolio with the higher expected return. It is worth noting that a significant challenge lies in identifying and constructing such portfolios. For instance, the efficient market hypothesis (Fama, 1970), even in its weak form, posits that analyzing past data may not be a reliable method for achieving abnormal returns.

It is important to recognize that the arbitrage mentioned in the previous paragraph considers the point estimates of the portfolio's expected return and beta, but does not include their confidence intervals. Overlooking the uncertainty associated with these parameters can have adverse consequences for investors. Black (1993) explored this issue of neglecting estimation errors in detail, suggesting that many anomalies identified in the investment literature may be the result of poor data analysis practices. Furthermore, Morettin and Bussab (2017) emphasize that relying solely on single-point estimates fails to provide insights into the magnitude of potential errors. They recommend constructing a confidence interval based on the distribution of these point estimates, which allows for a more comprehensive assessment of the data.

By considering the uncertainties associated with expected returns and betas, this study aims to provide valuable insights into the effectiveness of the zero-beta portfolio strategy. Understanding the potential risks and rewards of statistical arbitrage in light of these estimation errors is crucial for informed financial decision-making.

The objective of this study is to investigate the zero-beta portfolio strategy. Specifically, the study is interested in assessing a version of this strategy that considers not only the point estimates of certain parameters, but also the associated uncertainties in expected returns and betas. In addition, the study compares this approach to a zero-beta portfolio strategy that does not factor in these uncertainties. The expected stock return distribution is predicted using analysts' estimates and their deviations, while the betas and their uncertainty are calculated using Chen et al. (1986) multifactor pricing models based on the Kalman filter, a Bayesian approach to continuously estimate the state of a noisy system (Wells, 1996).

The study consists of five sections. Section 2 establishes the theoretical foundations for the development of the model and the methodology is presented in Section 3. Section 4 presents the results derived from the data and methodology outlined in Section 3. Lastly, Section 5 discusses the findings and outlines potential future research directions.

2 Theoretical foundations

2.1 Portfolio optimization under parameter uncertainty

According to Rockafellar and Wets (1991), many systems that require control or analysis involve uncertain parameters. They suggest that when faced with a probabilistic distribution of unknown parameters, it may be appropriate to consider stochastic models.

According to Bertsimas et al. (2011), robust and stochastic optimization approaches take uncertainty into account. From a portfolio optimization perspective, Bertsimas et al. (2011) suggest that mean-variance models that rely solely on the expected return point estimate could result in extreme allocations and prove sensitive to minor perturbations and parameter estimates. They further recommend incorporating uncertainty in expected returns into optimization methods to mitigate these challenges. Fan et al. (2014) indicate that from a valuation perspective, prices are only known to lie within an interval; therefore, robust optimization may be useful in modeling problems with multiple sources of uncertainty.

In finance, robust programming involves considering security prices, interest rates, exchange rates, and portfolio optimization (Xidonas et al., 2020). While traditional portfolio optimization methods have laid the groundwork for investment strategies, recent advances in portfolio theory suggest incorporating additional factors such as liquidity and uncertainty, as highlighted in the multiobjective approach proposed by Garcia et al. (2020). This can lead to more stable results compared to the classical approach. However, while there is a lot of literature on robust portfolios, there are few empirical studies that show the effectiveness of this method with real results (Xidonas et al., 2020). The authors suggest that additional empirical studies should be conducted to determine whether robust optimization yields better returns when tested in real-world scenarios.

As noted by Markowitz (1991), if an investor has precise knowledge of the returns of all stocks, they would logically choose to invest solely in the security offering the highest return, without any inclination to diversify. However, as Markowitz further argues, in a context marked by uncertainty, diversification becomes a rational and sensible strategy for investors. Maenhout (2004) supports this perspective by affirming that accounting for the uncertainty in expected returns is a prudent approach when making portfolio decisions. Furthermore, Goldfarb and Iyengar (2003) developed a model that incorporates parameter uncertainty that is applicable to both the classical mean-variance and value-at-risk approaches.

Fabozzi et al. (2009) present robust portfolio optimization considering mean-variance, value-at-risk, and conditional value-at-risk. They also note that when these models do not consider parameter uncertainty, they can suffer from data inadequacy.

The original portfolio optimization concept introduced by Markowitz (1952) laid the foundations for reliable portfolio optimization. However, the field has faced persistent challenges, including the sensitivity of portfolios to errors in parameter estimation, as well as the inherent difficulty in accurately forecasting future stock prices (Zhang et al., 2018; Kemaloglu Sibel et al., 2018). Addressing the issue of parameter estimation errors, Zhang et al. (2018) argue that portfolio optimization models typically treat parameter values as if they are unquestionably accurate, thereby overlooking the associated estimation errors. This oversight results in portfolios that are highly sensitive to asset selection. In response to this problem, Zhang et al. (2018) propose the implementation of robust techniques that take into consideration both the portfolio optimization model and the uncertainties surrounding parameter values.

According to Kolm et al. (2013), estimating expected returns in the context of modern portfolio theory poses a practical challenge, mainly because "risk-return optimization can be very sensitive to changes in inputs." The authors argue that relying solely on point estimates for parameters, without considering their associated uncertainties, may not be a wise approach. They also highlight that "recent portfolio optimization approaches have begun to account for the uncertainty surrounding expected returns and risk." To mitigate this problem, the authors point to several techniques, including Bayesian methods, the Black-Litterman approach, and robust optimization.

Finally, Fabozzi et al. (2009) present robust portfolio optimization considering mean-variance, value-at-risk, and conditional value-at-risk. The authors also state that these three models suffer from data inadequacy when they do not account for parameter uncertainty.

2.2 Statistical arbitrage considering parameter uncertainty

Statistical arbitrage is a self-financing investment strategy with a positive expected return and zero or near-zero expected risk (Caneo & Kristjanpoller, 2020). This process involves long and short investment strategies for assets with similar and particular characteristics (Caneo & Kristjanpoller, 2020). Kwan (1999) states that a long-short strategy aims to benefit investors by potentially profiting from undervalued and overvalued assets, which makes the investor's expected profit higher than a long-only investment.

Statistical arbitrage is a common strategy used by institutional investors, hedge funds, mutual funds, and proprietary trading firms (Zhao et al., 2019; Elliot et al., 2005). The primary objective is to capitalize on perceived market imbalances, with traders attempting to take full advantage of these, thereby driving prices toward a rational equilibrium (Göncü & Akyldirim, 2016; Do & Faff, 2010).

The profit in a statistical arbitrage process materializes when asset mispricing corrects itself in the future (Zhao et al., 2019; Bowen & Hutchinson, 2016). The disparity between the returns of the long and short portfolios is commonly referred to as the "spread" (Elliot et al., 2005; Kwan, 1999). Statistical arbitrage is recognized as a market-neutral strategy because it can hedge against systematic risk and its profitability is largely independent of market movements (Zhao et al., 2019; Bowen & Hutchinson, 2016; Elliot et al., 2005; Göncü & Akyldirim, 2016; Kwan, 1999). In the context of this study, the expected return of the portfolio is determined by the difference between the expected return of long positions and that of short positions.

A crucial point to consider, as asserted by Kwan (1999), is that achieving market neutrality does not necessarily require equal aggregate beta weights: this will offset the portfolio's overall market risk, making it independent of broader market movements. This concept, as demonstrated by Bodie et al. (2014), is illustrated in Equation 1:

$$\beta_z = w_s \beta_s - \left(1 - w_s\right) \beta_l = 0 \quad (1)$$

where:
β_z is the zero-beta portfolio;
w_s is the short portfolio weight;
β_s is the beta of the short portfolio;
β_s is the beta of the long portfolio.

From the parameter uncertainty perspective, Anish (2021) notes that the covariance of statistical arbitrage is a point estimate, and therefore this approach is susceptible to estimation errors: for this reason, the author claims that there is a strong need to work with more than a point estimate, and therefore the author developed a statistical arbitrage model that takes into account covariance uncertainty. According to Anish (2021), accounting for covariance uncertainty in portfolio optimization leads to robust weightings.

2.3 The risk-return models

Continuing with the statistical arbitrage strategy presented in the introduction to this study, there are two significant sets of parameters that are essential for forming both the long and short portfolios: expected return and risk factors.

2.3.1 *The expected return of securities*

In this study, the expected return of securities is determined based on analyst estimates. Furthermore, the variation in estimates for specific securities is regarded as the uncertainty associated with this parameter. These assertions are supported by Xue et al. (2019), Qin et al. (2016), Chen, Peng, Zhang and Rosyida (2017), Huang (2012), Bielstein and Hanauer (2019), Balakrishnan et al. (2021), Fernandes et al. (2012), Echterling et al. (2015), Zhai and Bai (2018), Xue et al. (2019), Chen et al. (2019), Fabozzi et al. (2009), Goetzmann and Massa (2008), and Verardo (2009).

2.3.2 *The factor models*

An important consideration is that achieving market neutrality may require more than a single-factor model. Bowen and Hutchinson (2016) employed a statistical arbitrage approach that used a multifactor model that considered market, size, value, momentum, and reversal factors. This multifactor model approach was also adopted by Caneo and Kristjanpoller (2020). It is important to clarify that this study's primary objective is not to devise a new risk-return model. Nevertheless, it is noteworthy that a well-constructed risk-return model can enhance the effectiveness of the statistical arbitrage process. Therefore, this section will provide a brief overview of this topic.

As stated by Fama and French (1997), a common problem in defining the cost of capital is the choice of model: the capital asset pricing models of Sharpe (1964) and Fama and French (1993) all face parameter standard errors. Campbell (1996) states that knowing how to measure the risk of an asset and what economic forces drive the additional reward an investor gets for bearing that risk are among the most fundamental questions in finance.

As mentioned by Elton and Gruber (1997), the single-index model with the market as a factor was the "earliest index model to receive wide attention," and it was first presented by Markowitz and later developed by Sharpe (1964) (Elton & Gruber, 1997). The authors state that the market model is as shown in Equation 2:

$$R_{it} = \propto_i + \beta_i R_{mt} + \varepsilon_{it} \quad (2)$$

where:
R_{it} is the return of asset i in period t;
$\propto_i$ is the exclusive expected return of asset i;
β_i is the market sensitivity of asset i;
R_{mt} is the market return in period t;
ε_{it} is the idiosyncratic risk of asset i in period t with zero mean and variance σ.

Elton and Gruber (1997) point out that the single-index model with the market as a factor has the following advantages: a low number of required estimates; the required inputs are easy for analysts to understand; and there is an increase in the accuracy of portfolio optimization compared to previous estimates. Galagedera (2007) notes that a large number of studies show that the CAPM cannot empirically explain stock returns, so other fundamental variables – such as size, book-to-market, macroeconomic variables, and price-to-earnings – have been incorporated into different models in an attempt to improve the single-factor model. However, the CAPM is still the most widely used pricing model by both practitioners and in the classroom (Jagannathan et al., 2010).

In addition, Elton and Gruber (1997) note that after the single-index market model was presented, many multi-index models have been published, the prototype of which is presented in Equation 3:

$$R_{it} = \propto_i + \sum_{j=1}^{J} \beta_i \, I_{jt} + \varepsilon_{it} \quad (3)$$

where:
β_i is the sensitivity of asset i to index I;
I_{jt} is the jth index;
J is the number of indexes.

Jagannathan et al. (2010) state that multifactor asset pricing models are particularly appealing to practitioners and risk managers because they are likely to provide a more detailed picture of the extent to which an asset is subject to different sources of risk. Elton and Gruber (1997) state that there are three types of multi-index structures: "1) market plus industry indexes; 2) surprises in basic economic indexes (e.g., production and inflation) (see Chen, Roll and Ross, 1986); and a portfolio of traded securities (e.g., an index of small minus large securities) (see Fama and French, 1993."

Avanidhar (2010) reviews the literature on the cross-sectional risk-return model and notes that there are more than fifty variables that have been used to explain asset returns. Avanidhar (2010), in line with Elton and Gruber's (1997) statement, then states that the multifactor risk-return models are sorted by style according to Fama and French (1993); factor based on macroeconomic influences, such as Chen et al. (1986); and the Connor and Korajczyk (1988, 1993) model.

Fama and French (1993) claim that the market beta alone cannot explain US common stock returns. In addition, the authors find that adding firm size and book-to-market to the market factor increases the explanatory power of the model. Fama and French (1996) assert that even though a large number of anomalies disappear when the three-factor model (Fama & French, 1993) is applied, it can also be assumed that some irrational pricing may still exist. Galagedera (2007) claims that Fama and French's (1993) multifactor model has been useful in explaining cross-sectional stock returns.

In addition to the three-factor model (Fama & French, 1993), Fama and French (2015) developed a five-factor model. According to Fama and French (2015), a five-factor model capturing size, value, profitability, and investment patterns performs better on average than the Fama and French (1993) three-factor model. Fama and French (2015) explain that the five-factor model is backed not only by empirical evidence but also by fundamental issues, which is supported by Miller and Modigliani's (1961) equation for the market value of a company.

Another important pricing model is the Arbitrage Pricing Theory proposed by Ross (1976), which takes into account the sensitivity of a stock/portfolio to unexpected news. Ehrhardt and Brigham (2019) demonstrate the APT equation in a very didactic way, as shown in Equation 4.

$$\overline{r}_i = \hat{r}_i + \left(\overline{F}_1 - \widehat{F}_1\right) b_{i1} + \ldots + \left(\overline{F}_j - \widehat{F}_j\right) b_{ij} + e_1 \quad (4)$$

where:
$\overline{r}_i$ is the realized rate of return of stock i;
$\hat{r}_i$ is the expected rate of return of stock i;
$\overline{F}_j$ is the realized value of economic factor j;
$\hat{F}_j$ is the expected value of factor j;
b_{ij} is the sensitivity of stock I to economic factor j;
e_1 is the effect of unique events on the realized return of stock i.

As can be seen from Equation 4, the APT does not indicate the sensitivity of a stock to the announcement of an economic factor, but rather its sensitivity to unexpected news: the realized value minus the expected value of a given factor. Chen, Roll, and Ross (1986) subsequently indicated the economic factors that could be consistent with the APT model: interest rate – long term minus short term, inflation, industrial production, and the spread between high-grade bonds and low-grade bonds

However, Galagedera (2007) affirms that there is no consensus on which model is best and that the search for a robust asset pricing model continues. In any case, this study will empirically test market-neutral portfolios under parameter uncertainty using the Chen et al. (1986) model.

In addition to the choice of models for implementing statistical arbitrage, another critical consideration is the set of assumptions used to compute these models. Regardless of the risk-return model chosen, a critical assumption is whether the factors are constant or subject to change over time.

By analyzing the characteristics of the problem presented in this paper, the Kalman filter may be a useful tool to solve this problem for beta estimation. The Kalman filter is a valuable tool for estimating time-varying market factors, as it effectively addresses the assumption that these factors are dynamic rather than static over time. This assumption is consistent with research by Jagannathan and Wang (1996) and Groenewold and Fraser (1999), which shows that financial factors, including asset betas, are not fixed but change in response to market conditions. Bramante and Gabbi (2006) also point to substantial evidence that asset betas vary over time, supporting the use of dynamic models such as the Kalman filter. When applied to market factor estimation, the Kalman filter has been shown to produce accurate results, especially in cases where traditional models such as GARCH are less effective. Asafo-Adjei et al. (2022) report that, based on forecast error metrics, the Kalman filter outperforms GARCH models in systematic risk estimation for twenty emerging markets.

Comparative studies also confirm the advantages of the Kalman filter in capturing time-varying factors. Mergner and Bulla (2008), for instance, assessed the time-varying beta of 18 sectors in Europe across several models and found that the Kalman filter provided superior estimates. Choudhry and Wu (2009) analyzed weekly beta estimates for UK firms and found that the Kalman filter approach produced more accurate results than three GARCH-based methods. Similarly, Mamaysky et al. (2008) demonstrated the superiority of the Kalman filter over the OLS model in beta forecasting, while Rajbhandary et al. (2013) found that it outperformed the moving window beta estimation. Grewal and Andrews (2014) note that the Kalman filter minimizes estimation error by reducing noise and other unmodeled variables through a quadratic error minimization function, allowing for accurate estimation in the presence of uncertain dynamics. This accuracy in capturing dynamic market behavior makes the Kalman filter an effective choice for optimal portfolio construction in settings with uncertain and evolving market factors.

3 Methodology

3.1 Combining parameter uncertainties

The excess return on the well-diversified portfolio of the single-factor model can be expressed as Equation 5.

$$R_p = E\left(R_p\right) + \beta_p * M \quad (5)$$

where R_p is the portfolio return, $E\left(R_p\right)$ is the expected portfolio return, β_p is the portfolio beta, and M is the market premium. In addition, $E\left(R_p\right)$ can be expressed as Equation 6.

$$E\left(R_p\right) = \sum w_i E(R_i) \quad (6)$$

For example, a portfolio composed of stocks of companies A and B, where company A has an expected return of 10% and represents 75% of the portfolio, while company B has an expected return of 5% and represents 25% of the portfolio, results in an expected portfolio return of 9%. β_p can be expressed as Equation 7.

$$\beta_p = \sum w_i \beta_i \quad (7)$$

Similarly, if Company A has a beta of 1.5 and Company B has a beta of 0.8, with Company A representing 75% of the portfolio and Company B representing 25%, the portfolio beta equals 1.127.

The return of an asset can be measured by the variation in the value of that asset in addition to the cash flow of that asset. When transaction costs are not considered, the expected return of a security is equal to the expected price and dividends of that security divided by its initial price minus 1 (Kwan, 1999).

Therefore, in this paper, the expected return of a security is considered to be the expected variation of the market capitalization of that security, given by the ratio of the analyst's market capitalization expectation to the actual market capitalization of the security at time t, plus the expected dividends of that company as set by the analysts. In addition, the parameter uncertainties are measured by the variance of all estimates (market cap and dividends).

Therefore, the expected excess return of an asset can be stated as shown in Equation 8[1].

$$R_{i_{t+1}} = \frac{\hat{P}}{P_o} + \frac{\hat{D}}{P_o} + \sum_{j=1}^{J} \beta_i \, I_{jt} \quad (8)$$

where $\hat{P}$ is the price target set by analysts and $\hat{D}$ is the dividend target set by analysts. Therefore, the expected return of an asset is given by the sum of its expected price and expected dividends divided by its current price, plus the sum of the market risk factors multiplied by their respective betas.

Fama and French (1997) state that the valuation of projects and firms is very imprecise due to uncertainty about either the risk factors or expected cash flows. The parameter uncertainties in this study are measured by the variance of all estimates, where $\hat{P}$ and $\hat{D}$ are

[1] Which is consistent with the APT model (Ross, 1976) – see Equation 4.

determined by the variance of the analysts' estimates and β_i is the uncertainty calculated by the Kalman filter.

Assuming that $\beta_i \; I_{jt}$ is a product of two Gaussian PDFs and independent of each other, this study will follow Smith's (2011) method in order to combine these curves. Equations 9 and 10 show the Gaussian equations for β_i and I_{jt}.

$$\beta_i = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(t-\mu_1)^2}{\sigma_1^2}} \tag{9}$$

$$I_{jt} = \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{(t-\mu_2)^2}{\sigma_2^2}} \tag{10}$$

So, following Smith's (2011) method, the mean of the product of $\beta_i \; I_{jt}$ is as shown in Equation 11:

$$\mu = \frac{\frac{\mu_1}{2\sigma_1^2} + \frac{\mu_2}{2\sigma_2^2}}{\frac{1}{2\sigma_1^2} + \frac{1}{2\sigma_2^2}} = \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2}{\sigma_2^2 + \sigma_1^2} \tag{11}$$

Moreover, based on Smith (2011), the variance of the product of $\beta_i \; I_{jt}$ is as shown in Equation 12:

$$\sigma^2 = \frac{1}{\frac{1}{1\sigma_1^2} + \frac{1}{\sigma_2^2}} = \frac{\sigma_2^2\sigma_1^2}{\sigma_2^2 + \sigma_1^2} \tag{12}$$

In other words, assuming that $\beta_i \; I_{jt}$ is a product of two Gaussian PDFs and independent of each other, the variance of the mean of this product can be calculated using Equation 11, while its variance is obtained by Equation 12. It is worth mentioning that I_{jt} represents the unexpected returns of macroeconomic factors, while β_i denotes the sensitivity of a security or portfolio to unexpected news (surprises) related to these specific macroeconomic factors.

Considering Equations 8, 11, and 12, the mean and variance of β_i are calculated from the Kalman filter, while the mean of I_{jt} is assumed to be zero[2] and the variance is set by the historical variance. Once the mean and variance of $\beta_i \; I_{jt}$ are calculated, it is still necessary to calculate the sum of the distribution curves of $\frac{\hat{P}}{P_o}$, $\frac{\hat{D}}{P_{o,}}$ and $\sum_{j \;\; 1}^{J} \; {}_{i} \, I_{jt}$ in order to find the distribution curve of $R_{i_{t+1}}$.

To do this, and assuming that the parameters are independent of each other, by applying the Normal Sum Theorem, the "mean and variance of a sum of statistically independent random variables is the sum of the means and variances of the individuals" (Lemons, 2002). Therefore, the point estimate of $R_{i_{t+1}}$ is the sum of the point estimates of $\frac{\hat{P}}{P_o}$ and $\frac{\hat{D}}{P_o}$, given that the point estimate of $\beta_i \; I_{jt}$ is 0, as claimed in the previous paragraph. Moreover, the variance of $R_{i_{t+1}}$ is the sum of the uncertainties of $\frac{\hat{P}}{P_o}$, $\frac{\hat{D}}{P_o}$ and the variances of $\beta_i \; I_{jt}$, as explained in Equation 12. In other words, the variance of a sum of statistically independent variables is equal to the sum of their variances, a principle known as the variance sum law. This concept is fundamental in statistics and is particularly relevant in various fields, including modeling (Pier-Olivier & Lamardelet, 2021; Bobkov et al., 2023).

In addition, the uncertainty of each portfolio – both long and short – is calculated as proposed by Markowitz (1952)[3]. Also, the uncertainty of the long and short portfolios combined is calculated as the product of the vector of portfolio weights[4] transposed by the portfolio variance-covariance matrix multiplied by the vector of portfolio weights. The optimal weights of each security and of the long and short portfolios are calculated by applying the decision criteria indicated in the following subsection.

Finally, as pointed out by Markowitz (1952), the expected return of a portfolio can be calculated as the weighted average of the expected return of each asset in that portfolio. Similarly, the portfolio beta of each factor will also be the weighted average of the respective asset's beta.

[2] The betas of the APT model are the sensitivity of a security/portfolio to unexpected news (surprise) about a specific macroeconomic factor. In this study, these surprises are assumed to have an expected variance but zero mean.

[3] By the product of the transposed vector of security weights and the security variance-covariance matrix multiplied by the vector of security weights.

[4] As mentioned before, the weights of the long and short portfolios do not necessarily need to be equal in order to create a market neutral portfolio.

3.2 The decision criteria

Considering the mean-variance approach first proposed by Markowitz (1952), Equation 13 shows that the optimal zero-beta portfolio in this study will be the one with the highest value of the expected return divided by its estimated variance, given that the individual betas of the long and short positions must satisfy Equation 1. In addition, the expected return of the portfolio will be determined by the difference between the expected return of the long position and the expected return of the short position, also called the expected spread.

$$\max \frac{w_l * Er_l - (1 - w_l) * Er_s}{estimated\ variance} \qquad (13)$$

where:

w_l is the weight of the long portfolio;

Er_l is the expected return of the long portfolio;

Er_s is the expected return of the short portfolio.

As in Markowitz (1952), the expected returns of the portfolios (either long or short) are determined by the weighted average of the expected returns of the individual assets. Also, to pursue Markowitz's (1952) maximum mean variance, the variance of each asset is equal to the sum of the combined uncertainties of its betas, market cap target, and dividends. Finally, this study makes a naive assumption for the covariance uncertainty that the parameter uncertainties between assets are independent.

The decision criterion of maximizing the ratio of the expected spread divided by the parameter uncertainties is supported by Kwan's (1999) study, which proposes a long-short optimization approach that maximizes the mean-variance ratio – similar to the tangency portfolio – for two portfolios with market neutrality. This strategy is also consistent with Göncü and Akyldirim's (2016) study, which considers not only the spread between two assets, but also the parameter uncertainties.

Given that the investor is concerned not only with maximizing their return but also with minimizing their risk, maximizing the ratio obtained by dividing the expected spread[5] by the parameter uncertainties is consistent with Markowitz (1952, 1991). Also, Kemaloglu Sibel et al. (2018) use two different approaches to apply robust optimization: the risk aversion formula based on the classical Markowitz formula, which maximizes the expected return for a given uncertainty, and the max-min, which minimizes the worst case scenario. It is worth mentioning that both approaches perform well according to the study of Kemaloglu Sibel et al. (2018).

3.3 Data and portfolio construction

All data such as market cap, target market cap, market cap estimate standard deviation, distributed dividends, expected dividends and their estimate standard deviations were collected from the Refinitiv Eikon database.

To calculate the betas using the Kalman filter, this study considered the weekly percentage change in the asset value. For the Chen et al. (1986) model, the following ETFs were used as proxies for the interest rate – long minus short term, inflation, industrial production, and the spread between high-grade bonds and low-grade bonds: iShares Short Treasury Bond ETF (SHV), iShares 20 Plus Year Treasury Bond ETF (TLT), Schwab US TIPS ETF[6] (SCHP), Vanguard Industrials Index Fund ETF (VIS), iShares iBoxx $ Inv Grade Corporate Bond ETF (LQD), and iShares iBoxx $ High Yield Corporate Bond ETF (HYG).

The initial beta estimate was calculated by applying a linear regression from January 18th of 2013 to December 27th of 2013. The resulting coefficients for each asset were then used as an estimate for 2014, and a new linear regression was calculated for each asset through 2014. This study then applies the Kalman filter to combine and estimate the betas for the following years, as well as their errors.

For the portfolio construction, even though the annual betas and their uncertainties were calculated considering the Chen et al. (1986) model of more than two thousand securities, due to computational limitations to optimally weight the assets, the portfolio considers only ninety-nine securities per year. In this study, the long and short portfolios were first built at the beginning of 2015 and then updated at the beginning of each subsequent year until 2022.

As already stated, the portfolio chosen for each year is the one with the highest value of the expected spread[7] divided by the combined uncertainties. Also, at

[5] The difference between the weighted expected return of the long and short portfolios.

[6] Index of the Bloomberg US Treasury Inflation Protected Securities (TIPS).

[7] The expected spread can be defined as the expected return on the long positions in the portfolio multiplied by the weight assigned to those long positions minus the expected return on the short positions in the portfolio multiplied by the weight assigned to those short positions.

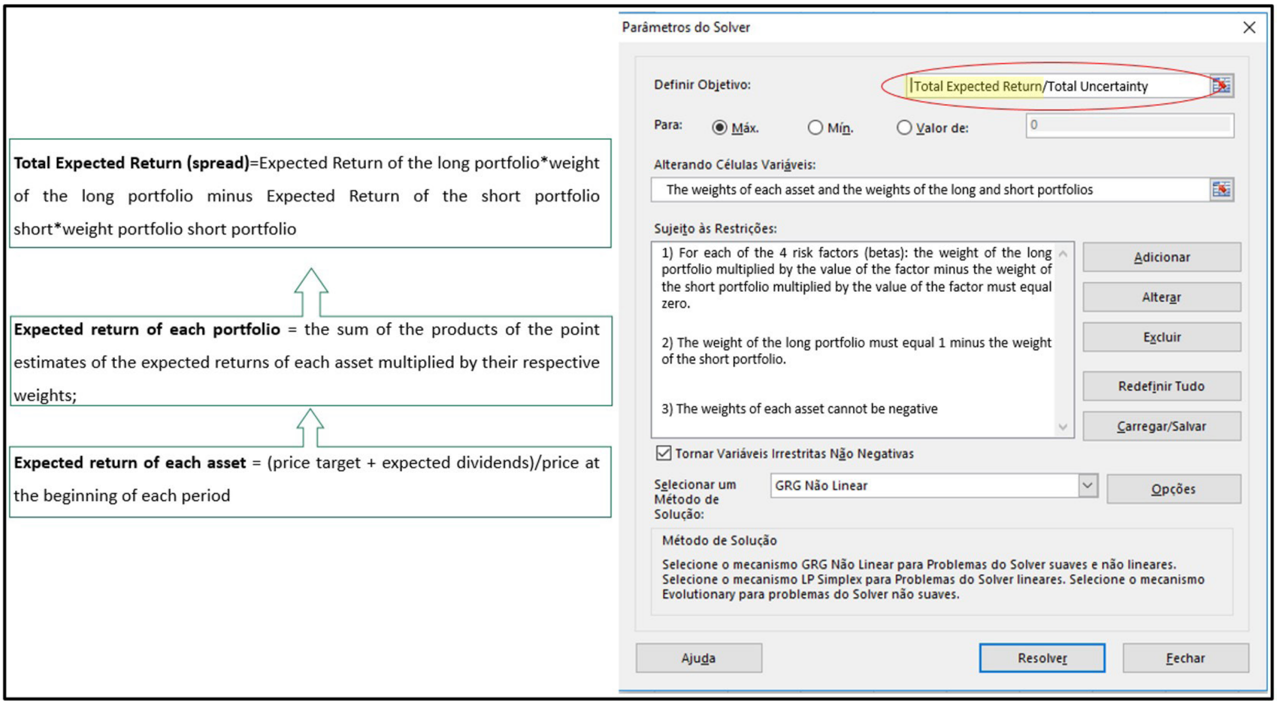

Figure 1. An illustration of the stochastic zero-beta portfolio optimization process applied in this study, as shown on the right, and how the expected return of each asset is integrated in order to obtain the total expected return of the zero-beta portfolio, as shown on the left

the end of each period, the portfolios are measured and compared with a portfolio that uses the same parameters but without considering their uncertainties (Supplementary Data 1 – Database).

Besides the objective of optimizing the ratio of the long-short expected return (spread) by the uncertainty, it is worth mentioning the other criteria applied:

a) For every risk factor[8], the weight of the long portfolio multiplied by the value of the factor minus the weight of the short portfolio multiplied by the factor must be equal to zero, in accordance with Equation 1.
b) The weight of the short portfolio is equal to one minus the weight of the long portfolio;
c) The sum of the weights of all securities – for each portfolio (long and short) – is equal to one;
d) The weight of all securities cannot be negative;
e) There are two variables that can be adjusted: the weights of the securities and the weight of the overall portfolio.

Figures 1 and 2 provide a more detailed description of the stochastic zero-beta portfolio optimization and how parameter uncertainties are incorporated into the optimization process.

Therefore, the portfolios are constructed by applying the following steps:

Step 1: Collect the data from the Refinitiv Eikon database;
Step 2: Calculate the betas of the companies and their uncertainties for the period 2015 to 2022, considering the macroeconomic factors[9] proposed by Chen et al. (1986) by applying the Kalman filter approach;
Step 3: Calculate the expected returns of the securities and their uncertainties using analysts' estimates of price targets and dividends;
Step 4: Combine the parameter uncertainties of the securities and the uncertainties of the portfolios;
Step 5: Model optimal zero-beta portfolios annually from 2015 to 2022 that aim to maximize the ratio of the expected spread divided by uncertainty, and

[8] Interest rate – long minus short term, inflation, industrial production, and the spread between high-grade bonds and low-grade bonds.

[9] Interest rate – long term minus short term, inflation, industrial production, and the spread between high-grade bond and low-grade bonds.

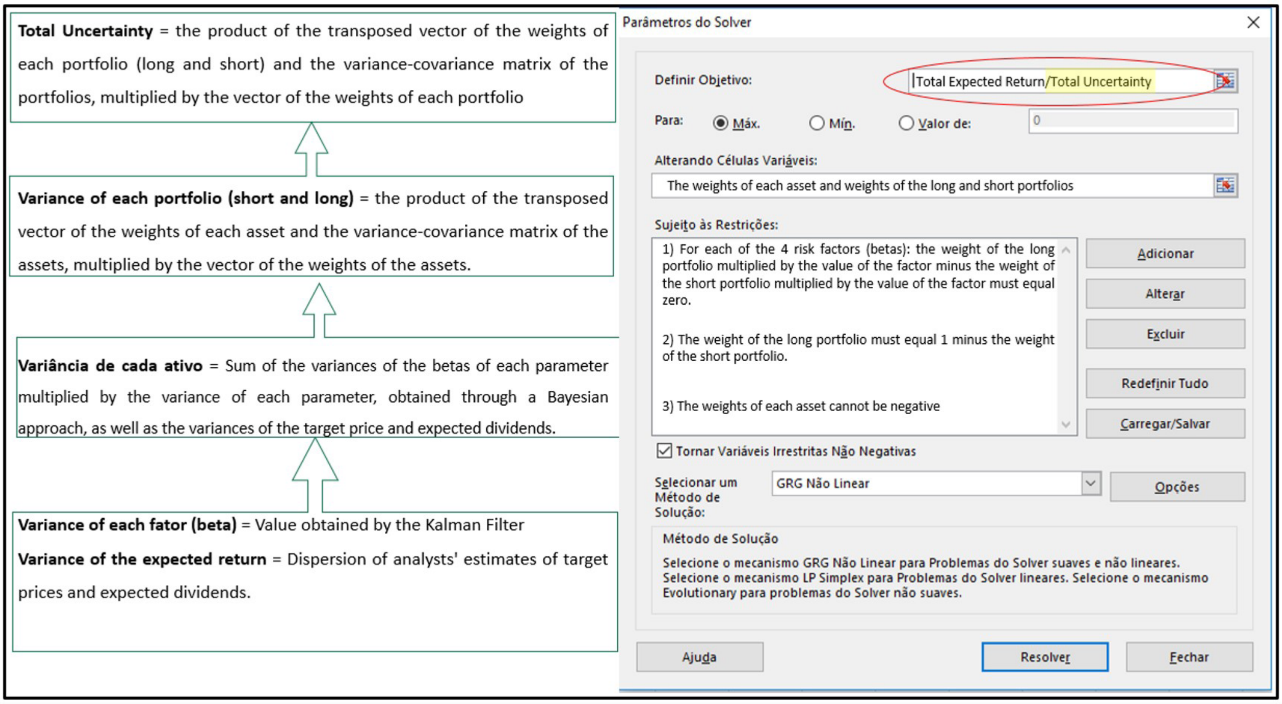

Figure 2. An illustration of the stochastic zero-beta portfolio optimization process applied in this study, as shown on the right, and how the parameter uncertainties are integrated into the optimization process in order to obtain the total uncertainty of the zero-beta portfolio, as shown on the left

others for the same period that maximize the expected spread but neglect uncertainty;

Step 6: Analyze the realized returns of the portfolios.

In addition, steps 2-6 were conducted entirely using Excel (Supplementary Data 1).

4 Results

The first and third columns of Table 1 show the portfolio's expected spread[10] after running the optimization tool with parameter uncertainties (first column) and without parameters uncertainties (third column). In addition, the second and fourth columns show the realized spread[11] of the respective portfolios. From now on, the zero-beta portfolios that maximized the ratio between the expected spread and the parameter uncertainties will be referred to as long-short stochastic portfolios, while the zero-beta portfolios that simply maximized the expected spread without considering the parameter uncertainties will be referred to as long-short normal portfolios.

As shown in Table 1, the actual returns realized each year differed significantly from the expected returns initially projected for the portfolio. In each case, the realized returns fell considerably short of the expected values, and in some cases, they even turned out to be negative. Several factors could have contributed to these disparities, including inaccurate forecasts of expected returns and the possible omission of other critical risk factors from the model. Regardless of the exact underlying causes, future research should delve deeper into the analysis of these differences to gain a more comprehensive understanding.

Table 1 presents results that favor the long-short stochastic portfolio over the long-short normal portfolio. In particular, the former shows significantly lower drawdowns, with negative returns of 0.17% and 3.58% in 2015 and 2016, respectively, compared to the latter, which experienced negative returns of 8.11% and 18.10% in the same years. In most years, the long-short stochastic portfolio consistently had smaller negative realized spreads compared to the long-short normal

[10] The expected spread can be defined as the expected return of the long positions in the portfolio multiplied by the weight assigned to those long positions minus the expected return of the short positions in the portfolio multiplied by the weight assigned to those short positions.

[11] The realized spread can be defined as the realized return of the long positions in the portfolio multiplied by the weight assigned to those long positions minus the realized return of the short positions in the portfolio multiplied by the weight assigned to those short positions.

Table 1
Portfolio Expected vs. Realized Spread Comparison (2015–2022). This table shows expected and realized spreads for both portfolio types, highlighting the reduced negative realized spreads in the stochastic portfolio, which generally indicates more resilient performance

Year	Long-short stochastic portfolio expected spread	Long-short stochastic portfolio realized spread	Long-short normal portfolio expected spread	Long-short normal portfolio realized spread
2015	13.24%	-0.17%	27.41%	-8.11%
2016	11.88%	-3.58%	79.57%	-18.10%
2017	9.57%	5.09%	43.65%	2.20%
2018	6.78%	2.24%	30.60%	-1.46%
2019	7.31%	-1.25%	70.57%	-3.39%
2020	9.35%	-2.03%	45.54%	-6.53%
2021	17.48%	3.56%	43.78%	-0.75%
2022	21.00%	-1.63%	44.62%	3.20%

portfolio. This indicates more stable performance with fewer extreme deviations between expected and realized spreads, suggesting that the stochastic approach may provide a more reliable return profile over time.

A significant result of this research concerns the computed root mean square error of both portfolios. The stochastic portfolio yielded a considerably smaller root mean square error, as shown in Table 2. As a result, when we compared the stochastic portfolio to the normal portfolio, we found that the stochastic portfolio had a substantially lower root mean square error – 11.34% compared to 52.47%. This indicates that the predictions for the stochastic portfolio were, on average, much closer to the actual results, suggesting that it may be a more accurate and reliable choice.

As expected, the long-short stochastic portfolios exhibit greater stability over time than the long-short normal portfolios. Specifically, the long-short normal portfolios tend to concentrate their investments in just a few securities, while the long-short stochastic portfolios are more diversified.

Table 3 compares the uncertainties of the long-short stochastic (column 2) and normal (column 3) portfolios over time with the realized standard deviations of the long-short stochastic (column 4) and normal (column 5) portfolios.

Since the long-short stochastic portfolio had a higher cumulative realized spread from 2015 to 2022 and a lower standard deviation than the long-short normal portfolio, the stochastic portfolio resulted in a higher ratio of realized spread to variance, as shown in Table 4. The total realized spread for the long-short stochastic

Table 2
Annualized root square error for the stochastic and normal portfolios.

Year	Root square error: Long-short stochastic portfolio	Root square error: Long-short normal portfolio
2015	13.41%	35.51%
2016	15.46%	97.66%
2017	9.57%	43.65%
2018	6.78%	30.60%
2019	7.31%	70.57%
2020	9.35%	45.54%
2021	17.48%	43.78%
2022	21.00%	44.62%

portfolio was 1.94%, excluding transaction costs, while the long-short normal portfolio had a realized spread of -29.9%, also excluding transaction costs. However, it is essential to include transaction costs to get a clearer picture of the net performance of the two strategies. For this, the study refers to Lesmond et al. (1999), who suggest an average transaction cost of 1.2% for large-cap companies.

Table 5 compares the two portfolios based on their Sharpe ratios. Using a 10-year constant maturity U.S. Treasury yield of 2.17% for January 1st of 2015 as the risk-free rate (Federal Reserve Bank of St. Louis, 2024), and factoring in the total realized spread after estimated transaction costs along with each portfolio's standard deviation, the long-short stochastic portfolio has a higher Sharpe ratio than the long-short normal portfolio.

Finally, Table 6 summarizes the key findings, particularly the advantages of the stochastic portfolio

Table 3
Uncertainties of the long-short portfolios (stochastic and normal) versus the realized standard deviations of the long-short portfolios (stochastic and normal).

Year	Uncertainty of the long-short stochastic portfolio model	Uncertainty of the long-short normal portfolio model	Stochastic zero-beta realized standard deviation per year	Non-stochastic zero-beta standard deviation per year
2015	0.000513149	0.099929997	0.15%	0.47%
2016	0.001614254	0.006214903	0.17%	0.48%
2017	0.002067432	0.003476831	0.19%	0.75%
2018	0.003172529	0.003700321	0.33%	0.49%
2019	0.002854613	0.001537929	0.21%	0.32%
2020	0.00197555	0.002860773	0.36%	1.29%
2021	0.002029139	0.000232843	0.26%	1.06%
2022	0.002052856	0.000859583	0.26%	0.51%

Table 4
Portfolio return and standard deviation.

Portfolio	Total realized spread minus estimated transaction costs	Standard deviation	Realized spread divided by variance
Long-short stochastic	0.74%	0.256%	1,135.94
Long-short normal	-31.10%	0.744%	(5,611.53)

Source: Prepared by the authors.

over the normal portfolio. It is important to emphasize that all results in this study were obtained using Excel to run the analysis (Supplementary Data 1).

5 Discussion and future studies

The findings of this study indicate that accounting for the uncertainties associated with parameter estimation in the construction of investment portfolios appears to yield significant benefits for investors, fund managers, and industry practitioners. Portfolios constructed using zero-beta models that incorporate parameter uncertainties consistently exhibited lower ex-post root mean square errors for each individual year analyzed, resulting in a lower overall root mean square error over the entire assessment period. These findings suggest that forecasting models for portfolio optimization that explicitly account for parameter uncertainties tend to produce more accurate ex-post predictions.

Moreover, portfolios that incorporate parameter uncertainties had lower ex-post drawdowns and standard deviations, indicating greater stability for investors, and higher Sharpe ratios, suggesting better risk-adjusted returns. Furthermore, the inclusion of parameter uncertainty considerations into portfolio construction led to higher realized returns compared to portfolios that did not incorporate such uncertainties. This unexpected outcome merits further comprehensive quantitative and qualitative investigation in future research endeavors.

Table 5
Sharpe ratio

Portfolio	Sharpe ratio
Long-short stochastic	(9.58)
Long-short normal	(44.69)

Table 6
Summary of key findings

Metric	Long-short stochastic portfolio	Long-short normal portfolio
Diversification	High	Low
Root mean square error	Lower	Higher
Sharpe ratio	Higher	Lower

The long-short stochastic portfolio formulated as part of this research showed an overall positive cumulative return over the period from 2015 to 2022. However, it is worth noting that there were specific years within this time frame where returns turned negative. These empirical findings are consistent with the assertions of Do and Faff

(2010), who claim that statistical arbitrage, which used to be a profitable strategy over a long time horizon, has experienced a declining trend in profitability. Consequently, investors, fund managers, and industry practitioners are advised to exercise caution and circumspection when considering the application of the statistical arbitrage strategy elucidated in this study.

As shown, the stochastic portfolios showed different performance from the non-stochastic portfolio in different years. This variation is largely due to the fact that the normal portfolio is composed of fewer assets, making it more susceptible to fluctuations in individual asset performance. This concentration increases its sensitivity to specific asset movements, resulting in greater variability in returns. Consequently, this sensitivity partly explains why the stochastic portfolios outperformed or underperformed the non-stochastic portfolio in certain years.

Throughout the process, computational limitations were a challenge. Although it was possible to calculate expected returns, market factors, and uncertainties for all US public companies, computational constraints required limiting the number of assets analyzed in the portfolio optimization process. To address this issue, the solution was to limit the portfolios by selecting only the 99 largest assets each year.

For subsequent research, the inclusion of additional risk factors, such as those proposed in the five-factor asset pricing model postulated by Fama and French (2015), could provide a more nuanced understanding of the risk-return profile of the strategy. Future studies could also apply multi-objective optimization when building optimal portfolios, as in the work of Yadav et al. (2023) and García et al. (2019). In addition, the empirical analysis presented in this study can be further complemented by the multi-period portfolio selection model, as applied by Wang et al. (2023), allowing for a more comprehensive evaluation of portfolio performance over time. Furthermore, extending the temporal scope of the analysis to encompass longer observation periods and diversifying the study to alternative financial markets, particularly those characterized as potentially less efficient than the US market, as recommended by Caneo and Kristjanpoller (2020), may yield valuable insights into the robustness and adaptability of the strategy.

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SUPPLEMENTARY MATERIAL

Supplementary Data 1 – Database

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