RBGN REVISTA BRASILEIRA DE GESTÃO DE NEGÓCIOS
REVIEW OF BUSINESS MANAGEMENT
© FECAP
ISSN 1806-4892

Received on
November 17, 2014
Approved on
November 14, 2015

1. Leticia de Almeida Costa
Doctor of Science in Production Engineering
Pontifical Catholic Univesity of Rio de Janeiro
(Brazil)
[leticiaalmeidacosta@gmail.com]

2. Felipe Philadelpho Azevedo
Master of Science in Production Engineering
Pontifical Catholic Univesity of Rio de Janeiro
(Brazil)
[felipephiladelph@gmail.com]

3. Carlos Patricio Samanez
Doctor of Science in Business Administration
Fundação Getúlio Vargas – FGV-SP
(Brazil)
[cps@puc-rio.br]

Review of Business Management

DOI:10.7819/rbgn.v17i57.2253

Investment strategies in the Brazilian industry of aluminum cans: an analysis in the context of real options games

Leticia de Almeida Costa,
Felipe Philadelpho Azevedo
Department of Industrial Engineering Pontifícia Universidade Católica do Rio de Janeiro, RJ, Brazil

Carlos Patrício Samanez
Department of Industrial Engineering, Pontifícia Universidade Católica do Rio de Janeiro, and Faculty of Economics, Universidade do Estado do Rio de Janeiro, RJ, Brazil.

Responsible editor: Andre Taue Saito, Dr.
Evaluation process: Double Blind Review

ABSTRACT
Objective – The purpose of this article is to analyze the impact of preemption (first move advantage, with the consequent emergence of negative externalities to competitors) in situations that consider the optimal moment for investment, in the context of asymmetric oligopolies and using options games method.

Design/methodology/approach – The developed model was applied to the Brazilian aluminum can industry, in which three firms made up an asymmetric oligopoly, allowing strategic interactions and their consequences on firms' investment decisions to be analyzed.

Findings – In situations of preemption, the results show the relevance of using a dynamic model, allowing us to observe the importance of obtaining a competitive advantage in cost, and showing that it is possible to obtain monopoly profits or take advantage of isolated expansion for a longer period. If this advantage is great, rivals' threat of preemption can be considered irrelevant, and that the firm will invest in monopolistic time, ignoring the possibility of rivals' entry.

Practical implications – In a competitive environment, firms need to decide whether the best strategy is to invest earlier, acquiring a competitive advantage over their rivals, or to postpone their investments, to acquire more information and mitigate the eventual consequences of market uncertainties. This work shows how to do it.

Originality/value – This is the first work that, by applying real options games, studies the impact of preemption of investment in oligopolistic asymmetric environment in the Brazilian industry of aluminum cans.

Keywords – Real options games, Asymmetric oligopoly, Investment under uncertainty.

1 INTRODUCTION

The most traditional methodology in capital budgeting is the discounted cash flow (DCF) method – which is analysis of the feasibility of an investment project based on expected future cash flows, and on analysis of its main index, the also traditional Net Present Value (NPV). There are several disadvantages which make the use of this index inappropriate in certain cases. Firstly, because NPV analyzes investment opportunities as “now or never” decisions, under passive management, without considering the flexibilities that are inherent to most capital investments, which is not very realistic, and ignores the consequences of possible actions by competitors (Samanez, Ferreira, Nascimento, Costa & Bisso, 2014).

Due to increasingly sophisticated technologies, and to markets becoming more and more dynamic and competitive, the value of flexibilities inherent to capital investment projects must be considered in analysis of its economic feasibility. As opposed to the traditional approach, the methodology known as Real Options considers the dynamic nature and the flexibilities involved in decision-making processes. However, this method presumes that investment decisions are made in an individual way, ignoring possible competitive endogenous interactions. A third methodology, the Game Theory, when applied to investment decisions, describes and anticipates rational behavior in environments wherein individuals are independent and interact with each other. In this situation, the actions of a firm have an effect on rival firms, and the latter, in turn, have an impact on the former. When the analysis is applied in a context of imperfect competition, such as is the case in oligopolies, a small number of firms with similar interests interact in such a way that their actions may influence of each other’s profits and individual values. Although in this methodology the effects of competition and opportunity for cooperation are modelled endogenously, the Game Theory fails to explain why firms must have incentives to remain flexible so as to be able to react to market uncertainties.

Games with real options are, as such, the integrated solution used to overcome the deficiencies of the abovementioned methodologies. The method was developed to capture the dynamics of strategic interactions in a competitive and uncertain environment, and is capable of guiding managerial decisions, enabling a more complete quantification of market opportunities, since it evaluates the sensitivity of strategic decisions to exogenous variables and competitive situations (Grenadier, 2002).

In academic literature, there are presently studies that analyze the relationship between managerial flexibility and competitive strategy through analysis of real options and game theory. Among them, one may highlight the model developed by Smit and Ankum (1993), wherein the authors analyze a game of perfect equilibrium for investment making decisions in one or two stages for two firms. Smit and Trigeorgis (2009) have analyzed a duopoly competition, and Huisman (2001) and another study have assumed in their studies that two firms may simultaneously make decisions under demand uncertainty and competition (Huisman, Kort, Pawlina and Thijssen, 2005). In another study, Smit and Trigeorgis (2004) have illustrated the use of real options and game theory to analyze investment opportunities in case of competitive strategy decisions under uncertainty.

Costa and Samanez (2008) presented a mathematical model based on real options theory and on game theory applied to the residential real estate market in the city of Rio de Janeiro, Brazil. The central idea of the model is the search for the balance between the demand and supply of residential units in a particular region, considering the effect on the prices of these assets and on the attitude of competitors in this market.

Thijssen, Huisman and Kort (2012), using games with options, dealt with the case of symmetrical equilibrium strategies for a duopoly. They revealed that, when both firms claim to be the first to invest, coordination problems begin to emerge. To solve this problem, the authors suggest the use of a method that involves employing mixed symmetric strategies based on Fudenberg and Tirole's model (1985), who discussed mixed strategies in the deterministic case. They reveal that, in many cases, it is incorrect to state that, in balance, the probability of both firms investing simultaneously, when investing is good for only one of them is zero. Huisman and Kort (1999) had already demonstrated this in the context of symmetric firms, and Dias and Teixeira (2010) described this result in the symmetric case, also showing the necessary conditions for the asymmetric case.

Using games with options, Angelou and Economides (2009) modeled the competitive interactions that emerge in the sector of communications and information technology. The model developed was applied to a real case study, showing that the problem can be formulated and solved by the applied approach.

The main purpose of this article is to analyze the impact of preemption (first move advantage, with the consequent emergence of negative externalities to competitors) in situations that consider the optimal moment for investment, in the context of asymmetric oligopolies and using options game method. The developed model was applied to the Brazilian aluminum can industry, in which three firms made up an asymmetric oligopoly, allowing strategic interactions and their consequences on firms' investment decisions to be analyzed.

In situations of preemption, article results show the relevance of using a dynamic model in subject modeling, allowing us to observe the importance of obtaining a competitive advantage in cost, and showing that it is possible to obtain monopoly profits or take advantage of isolated expansion for a longer period. The results also show that, if competitive advantage is very great, rivals' threat of preemption can be considered irrelevant, and that the company will invest in monopolistic time, ignoring the possibility of rivals' entry.

This paper is divided as follows. Section 2 presents the concepts of options games. Section 3 presents the Brazilian aluminum can industry. Section 4 develops the options games model for asymmetric oligopolies. Section 5 applies the proposed model to the market of aluminum cans in Brazil, and presents the results of a sensitivity analysis to various parameters, and section 6 presents the paper's conclusions.

2 REAL OPTIONS GAMES

The optimal moment for investment is associated to the choice of the ideal moment to make an investment. How should one decide between investing now, later on, or abandoning a project? A firm in a competitive environment may have an optimal investment policy that is completely different from a monopolistic firm. In situations in which there is the possibility of competitors anticipating their rival's actions, the value of postponing investments deteriorates and its value decreases. According to Chevalier-Roignant and Trigeorgis (2011), the presence of competitors generally leads firms to invest earlier than in the monopolistic case.

In a competitive environment, firms need to decide whether the best strategy is to invest earlier, acquiring a competitive advantage over their rivals, or to postpone their investments, to acquire more information and mitigate the eventual consequences of market uncertainties. Exogenous uncertainties, in the form of fluctuating demand, prices or production costs, may inhibit a firm's investments. In contrast, facing presence of pressures or threats from rival firms, it may create a competitive advantage if it is the first to invest. This trade-off may be examined through a combination of game theory with real options.

3 THE BRAZILIAN ALUMINUM CAN INDUSTRY

According to the Brazilian Association of Manufacturers of Highly Recyclable Cans (Associação Brasileira dos Fabricantes de Latas de Alta Reciclabilidade/Abralatas) (http://www.

abralatas.org.br/), Rexam PLC, Crown Co. and Latapack-Ball Corporation are three firms in Brazil with an approximate production capacity of 28 billion units per year. The industry as a whole is characterized by high entry barriers, since the economies of scale enabled by many factories favor large producers. Shipping costs are substantial and, therefore, a key factor of competitive advantage is the proximity of factories to places where clients obtain supplies.

Aluminum cans are used to package beverages, both alcoholic and non-alcoholic. Based on the annual reports of Rexam (2012), of Crown Holdings Inc. (2012) and of the Latapack-Ball Corporation (2012), the main products that use aluminum cans are beer and soda, which together, in 2012, added up to 76% of the can consumption all over the world. Other products with smaller participation are ice tea, coffee, juices and energy drinks, to name a few. The increase in preference for can results from several factors, such as: practical and easy transportation and refrigeration, respect for the environment, and the fact that cans are recycled rapidly and economically. In 2010, Brazil became the third largest beer producer in the world, surpassing Germany and Russia, and second only to China and the USA. The exponential increase of consumption in the northeast of Brazil and the increase in per capita income were the main drivers of this growth. Moreover, the creation of stricter traffic legislation led to the migration of beer consumption from bars (where glass bottles are predominant) to homes, boosting sales of cans in supermarkets. Over the next few years, international events such as the 2016 Olympic Games will further stimulate the consumption of aluminum cans in Brazil.

4 THE OPTIONS GAMES MODEL – ASYMMETRIC OLIGOPOLY

Considering the Brazilian industry made up of three oligopolistic firms of different sizes, competing in a market where there is a preemption threat and firms are not homogeneous (for the same investment, a firm has lower operating costs than another), this section, in the context of games with real options, equations and analytical solutions that govern the investment decision-making process for each of the three firms will be presented.

In the case applied to the Brazilian aluminum can industry, we consider three firms. One firm with a low cost (*l*), another with intermediate cost (*int*), and the last with high cost (*h*), in such a way that $c_h > c_{int} > c_l$. All of these firms operate in the market and have the option to expand their can production, making a new investment (I), the same for all the firms. However, since their operational costs differ, the oligopoly is asymmetric, and there is, thus, a competitive advantage for the company with the smallest operational cost. When one of the firms decides to expand, the profit of the others is altered, that is, the model considers negative externalities.

Considering market uncertainties, the proposed model will suppose that the firm's profit comprises two parts: a determining part, representing the firm's installed capacity (present and future), and is represented by $\overline{\pi}_i$ (where *i* is equal to *l*, *int* or *h*), and a stochastic multiplicative shock, $\tilde{X}_t$, which represents industry uncertainty. We presumed that this shock follows a geometric brownian motion (GBM) process represented by equation (1) (Dixit & Pindyck, 1994).

$$d\tilde{X}_t = \alpha\tilde{X}_t dt + \sigma\tilde{X}_t dz_t \qquad (1)$$

where is the tendency, σ is the volatility, and dz_t is the increment of Wiener given by equation (2).

$$dz_t = \varepsilon\, dt \text{, with } \varepsilon \sim N(0,1) \qquad (2)$$

where $N(0,1)$ is a standardized Normal distribution (average zero, variance 1).

Thus, the profit of organization *i* is given by equation (3), as follows:

$$\tilde{X}_t \cdot \bar{\pi}_i^n = \tilde{\pi}_i^n \qquad (3)$$

where n is the number of firms that have already carried out the option of market expansion.

When a firm carried out its option of expansion (investing I), this affects the other firms' profits, since the model considers the possibility of negative externality in prices, given by the demand curve (an increase in quantity produced reduces the final product price).

The Leader (L) shall be defined as the sole firm that carried out its option for expansion. The natural order, due to competitive advantage, is that the firm with lower costs becomes the Leader (L_l). Under certain conditions, the high-cost firm coming before the low-cost one can be subgame-perfect Nash equilibrium (SPNE), as shown by Dias and Teixeira (2010) in the context of asymmetric duopoly. But this article will only address the most intuitive SPNE.

When a firm invests after the Leader, it becomes the First Follower. Following the same reasoning, the firm with intermediate costs shall be the second to invest (F_{int}). If two firms have already expanded their production and a third firm decides to invest, that firm shall be the Second Follower. Furthermore, according to the natural order, it must be the firm with higher costs (FF_h). Table 1 relates the deterministic profit flows of each of the three firms with the four possible stages of industry development.

TABLE 1 – Oligopoly profit in four stages of development

		Fluxo de lucro determinístico		
Tempo	**Estrutura da Indústria**	**Firma *L***	**Firma *F***	**Firma *FF***
$t < \tilde{T}_L$	Ninguém investe	$\bar{\pi}_L^0$	$\bar{\pi}_F^0$	$\bar{\pi}_{FF}^0$
$\tilde{T}_L \leq t \leq \tilde{T}_F$	Apenas uma investe	$\bar{\pi}_L^1$	$\bar{\pi}_F^1$	$\bar{\pi}_{FF}^1$
$\tilde{T}_F \leq t \leq \tilde{T}_{FF}$	Duas investem	$\bar{\pi}_L^2$	$\bar{\pi}_F^2$	$\bar{\pi}_{FF}^2$
$t \geq \tilde{T}_{FF}$	Todas investem	$\bar{\pi}_L^3$	$\bar{\pi}_F^3$	$\bar{\pi}_{FF}^3$

The solution can be found through backwards induction. In this approach, first it is necessary to estimate the value at the node of the final decision, in our case the trigger (the value of the firm that sets off the option to invest) of the Second Follower, to then calculate the trigger of the First Follower and, finally, the trigger for the Leader. According to Chevalier-Roignant and Trigeorgis (2011), the expected present value of the Second Follower firm (in $t_0 = 0$) investing in expansion over time $\tilde{T}_{FF}$, using the concepts of expected value for the stochastic discount factor (Dias, 2015; Dixit & Pindyck, 1994), may be defined by equation (4):

$$FF_0(X_T) = \frac{X_0 \cdot \bar{\pi}_{FF}^0}{\delta} + B_0(\tilde{T}_L) \cdot \frac{X_L[\bar{\pi}_{FF}^1 - \bar{\pi}_{FF}^0]}{\delta} + B_0(\tilde{T}_F) \cdot \frac{X_F[\bar{\pi}_{FF}^2 - \bar{\pi}_{FF}^1]}{\delta} + B_0(\tilde{T}_{FF}) \cdot \left[\frac{X_{FF}[\bar{\pi}_{FF}^3 - \bar{\pi}_{FF}^2]}{\delta} - I\right] \qquad (4)$$

where $B_t(\tilde{T})$ is the expected value of the stochastic discount factor, and δ is the rate of distribution of the firm's dividends. The values above are for the case in which the initial stochastic shock (X_0) is lower than the first trigger (X_L), which is one of the assumptions that we will use in the model.

The firm that invests last maximizes its value by selecting the optimal moment for investing, that is, selecting the trigger over time $\widetilde{T}_{FF}$. As per the first order condition, we take the partial derivative of FF_0 in relation to the stochastic variable X_{FF}, equalizing the result with zero and obtaining the following equation (Chevalier-Roignant and Trigeorgis, 2011):

$$B_X(X_{FF}) \cdot \left[\frac{X_{FF}[\bar{\pi}_{FF}^3 - \bar{\pi}_{FF}^2]}{\delta} - I\right] + B_0(X_{FF}) \cdot \left[\frac{[\bar{\pi}_{FF}^3 - \bar{\pi}_{FF}^2]}{\delta}\right] = 0 \quad (5)$$

$$B_X(\cdot) = \frac{\partial B}{\partial X_{T_{FF}}} \quad (6)$$

$$B_0(X_{FF}) = \left(\frac{X_0}{X_{FF}}\right)^{\beta_1} \quad (7)$$

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} \quad (8)$$

The stochastic discount factor, which used to be a function of time, is now a function of value X_{FF} (there are two equivalent strategies: the investor seeks the optimal moment to make his decision, or the optimal target level for investing – value of the stochastic shock) (Dias, 2015; Dixit and Pindyck, 1994).

The derivative of the expected value of the discount factor in relation to X_{FF} is:

$$B_X(X_{FF}) = -\beta_1 \frac{X_0^{\beta_1}}{X_{FF}^{\beta_1+1}} \quad (9)$$

Replacing equations (7) and (9) in equation (5), it is then possible to find the value for the trigger (X_{FF}) of the Second Follower firm, given by the following equation (10):

$$X_{FF} = \frac{\beta_1}{\beta_1 - 1} \cdot \frac{\delta \cdot I}{\bar{\pi}_{FF}^3 - \bar{\pi}_{FF}^2} \quad (10)$$

The expected present value of the First Follower firm (over time $t_0 = 0$) investing in expansion at trigger $\widetilde{T}_F$, for $X_0 \leq X_{T_L}$, is given by equation (11):

$$F_0(X_T) = \frac{X_0 \cdot \bar{\pi}_F^0}{\delta} + B_0(\tilde{T}_L) \cdot \frac{X_L[\bar{\pi}_F^1 - \bar{\pi}_F^0]}{\delta} + B_0(\tilde{T}_F) \cdot \left[\frac{X_F[\bar{\pi}_F^2 - \bar{\pi}_F^1]}{\delta} - I\right] + B_0(\tilde{T}_{FF}) \cdot \frac{X_{FF}[\bar{\pi}_F^3 - \bar{\pi}_F^2]}{\delta} \quad (11)$$

If there was no threat of preemption, the First Follower (F_{int}) would invest at the trigger point that would maximize the firm's value. However, due to the threat of preemption, the possibility exists where it may not wait as long to invest. The Second Follower (FF_h, for being the third to invest) may have an incentive to become the First Follower (F_h). If the value of the firm as First Follower is greater than its value as the Second Follower, that is, if $F_h > FF_h$. Therefore, the strategy of the First Follower firm (F_{int}) will depend on the size of its competitive advantage. If this advantage is big enough, it may therefore not have to worry about preemption, which occurs when the Second Follower firm (FF_h) is never better off as First Follower (F_h). In this case, the First Follower firm (F_{int}) shall invest by choosing its trigger in a monopolistic manner, which shall be designated by X_F^M. In this context, using the first order condition to maximize the firm's value, one must take the partial derivative of F_0 in relation to stochastic variable X_F^M, equalizing the result to zero and obtaining the following equation:

$$B_X(X_F^M) \cdot \left[V_{T_F}[\tilde{\pi}_F^2 - \tilde{\pi}_F^1] - I\right] + B_0(X_F) \cdot V_X[\tilde{\pi}_F^2 - \tilde{\pi}_F^1] = 0 \quad (12)$$

The value for the trigger X_F^M without the threat of preemption shall then be:

$$X_F^M = \frac{\beta_1}{\beta_1 - 1} \cdot \frac{\delta \cdot I}{\bar{\pi}_F^2 - \bar{\pi}_F^1} \quad (13)$$

In case the competitive advantage is small, there may be a region in which the Second Follower firm (FF_h) will be better off investing as the First Follower (F_h). In this scenario, the optimal strategy of the second firm, to avoid preemption, is to invest when the value of the Second Follower (FF_h) is equal to its value as First Follower (FF_h), that is, when $F_h = FF_h$. Therefore, the trigger for preemption of the First Follower firm (F_h) is given by:

$$X_F^P = \inf\{\tilde{X}_t < X_{FF} \,|\, F_h(\tilde{X}_t) = FF_h(\tilde{X}_t)\} \quad (14)$$

The trigger for the First Follower firm will be the minimum value between the trigger calculated under monopolistic competition and the preemption trigger:

$$X_F = \min\{X_F^M; X_F^P\} \quad (15)$$

According to Chevalier-Roignant and Trigeorgis (2011), the expected present value of the leading firm (at time $t_0 = 0$) investing in expansion at the trigger $\tilde{T}_L$ is:

$$\begin{aligned} L_0(X_T) &= \frac{X_0 \cdot \bar{\pi}_L^0}{\delta} + B_0(\tilde{T}_L) \cdot \left[\frac{X_L[\bar{\pi}_L^1 - \bar{\pi}_L^0]}{\delta} - I\right] \\ &+ B_0(\tilde{T}_F) \cdot \frac{X_F[\bar{\pi}_L^2 - \bar{\pi}_L^1]}{\delta} \\ &+ B_0(\tilde{T}_{FF}) \cdot \frac{X_{FF}[\bar{\pi}_L^3 - \bar{\pi}_L^2]}{\delta} \end{aligned} \quad (16)$$

Since the value of the firm depends on the initial state of demand, the following cases are possible:

- For $X_L \leq X_0 < X_F$, only the Leader firm invests immediately:

$$\begin{aligned} L_0(X_T) &= \frac{X_0 \cdot \bar{\pi}_L^1}{\delta} - I + B_0(\tilde{T}_F) \cdot \frac{X_F[\bar{\pi}_L^2 - \bar{\pi}_L^1]}{\delta} \\ &+ B_0(\tilde{T}_{FF}) \cdot \frac{X_{FF}[\bar{\pi}_L^3 - \bar{\pi}_L^2]}{\delta} \end{aligned} \quad (17)$$

- For $X_F \leq X_0 < X_{FF}$, the Leader firm and First Follower firm invest over time t=0:

$$L_0(X_T) = \frac{X_0 \cdot \bar{\pi}_L^2}{\delta} - I + B_0(\tilde{T}_{FF}) \cdot \frac{X_{FF}[\bar{\pi}_L^3 - \bar{\pi}_L^2]}{\delta} \quad (18)$$

- For $X_0 \geq X_{FF}$, all three firms invest immediately:

$$L_0(X_T) = \frac{X_0 \cdot \bar{\pi}_L^3}{\delta} - I \quad (19)$$

The same reasoning used in analyzing the trigger X_F shall be used for the Leader firm. In the absence of the preemption threat, the Leader firm (in this case L_l) would invest in the trigger that would maximize the firm's value. However, due to preemption threat, the possibility exists that it cannot wait that long to invest. The First Follower may have incentives to become Leader (L_{int}). This will occur if the value of that firm as the Leader is greater than its value as the First Follower, that is, if $L_{int} > F_{int}$.

The strategy of the Leader firm (L_l) shall depend on the size of its competitive advantage. If the advantage is great enough, it may not have to worry about preemption. Thus, it will occur whenever the First Follower firm (F_{int}) is never better off as the Leader (L_{int}). In this case, the Leader firm (L_l) shall invest by choosing its trigger in a monopolistic manner, which shall be called X_L^M. In this case, using the first order condition to maximize the firm's value, the partial derivative of L_0 must be taken, in relation to the variable of stochastic X_L^M and equal the result to zero (Chevalier-Roignant and Trigeorgis, 2011). Proceeding accordingly, the value for the trigger X_L^M without the threat of preemption shall be given by the following equation:

$$X_L^M = \frac{\beta_1}{\beta_1 - 1} \cdot \frac{\delta \cdot I}{\bar{\pi}_L^1 - \bar{\pi}_L^0} \quad (20)$$

In case the competitive advantage is small, there may be an area wherein the First Follower (F_{int}) is better off investing as the Leader (L_{int}). In this scenario, the optimal strategy of the first firm, to avoid preemption, is to invest when the value of the First Follower (F_{int}) is equal to its value as the Leader (L_{int}), that is, $L_{int} = F_{int}$. Therefore, the trigger for preemption of the Leader firm (L_l) shall be given by:

$$X_L^P = \inf\{\tilde{X}_t < X_F \,|\, L_{int}(\tilde{X}_t) = F_{int}(\tilde{X}_t)\} \quad (21)$$

The trigger of the Leader firm shall be the minimum value between the trigger calculated in a monopolistic manner and the preemption trigger:

$$X_L = \min\{X_L^M; X_L^P\} \quad (22)$$

The firms shall compete in the form of a Cournot oligopoly. In a market, with *n* oligopolistic firms competing in quantity, producing a homogenous good, where each firm *i* has a marginal cost of production c_i, the demand shall be represented by the linear function given by the equation (23).

$$P(Q_T) = a - bQ_T, \text{com} a > 0, b > 0 \text{ e} a > bQ_T \quad (23)$$

The profit function of firm is given by:

$$\pi_i(q_i, Q_{-i}) = [P(q_i, Q_{-i}) - c_i]q_i, \quad para i = 1, \ldots, n \quad (24)$$

where $Q - i$ gives us the total quantity produced by all the firms in the market except for firm *i*, with $Q_T = q_i + Q - i$. To obtain the Cournot–Nash equilibrium, one must find the profile for the strategy for optimal production $(q_i^C, q_{i+1}^C, \ldots, q_n^C)$ in such a manner that each firm *i* maximizes its profit considering their rivals' choices as a given. This leads us to the following first order condition:

$$\frac{\partial \pi_i(q_i^C, Q_{-i}^C)}{\partial q_i} = 0 \quad (25)$$

$$q_i^C \cdot \frac{\partial P(q_i^C, Q_{-i}^C)}{\partial q_i} + P(q_i^C, Q_{-i}^C) = c_i \quad (26)$$

In the Cournot oligopoly model, with asymmetric firms, we must solve the following system of *n* equations and *n* variables:

$$\begin{cases} a - 2bq_i^C - b(Q_T^C - q_i^C) = c_i \\ a - 2bq_{i+1}^C - b(Q_T^C - q_{i+1}^C) = c_{i+1} \\ \ldots \\ a - 2bq_n^C - b(Q_T^C - q_n^C) = c_n \end{cases} \quad (27)$$

Solving the system, one finds the total quantity produced in this market:

$$Q_T^C = \left(\frac{n}{n+1}\right)\left(\frac{a - \bar{c}}{b}\right) \quad (28)$$

where $\bar{c} \equiv \sum_{j=1}^{n} c_j / n$ gives us the average cost of production for the industry.

The individual quantity of each firm is obtained by replacing the industry's total quantity in each equation of the system (27):

$$q_i^C = \frac{1}{n+1}\left(\frac{a - nc_i + (n-1)\bar{c}_{-i}}{b}\right) \quad (29)$$

The equilibrium price shall be:

$$P^C = \bar{c} + \frac{a - \bar{c}}{n+1} \quad (30)$$

The profit of firm *i* shall be given by:

$$\pi_i^C = \frac{1}{(n+1)^2} \frac{(a - nc_i + (n-1)\bar{c}_{-i})^2}{b} \quad (31)$$

where $\bar{c}_{-i} \equiv \sum_{j \neq i}^{n} \frac{c_j}{n-1}$ is the average production costs of all market firms, except for firm *i*.

5 APPLYING THE MODEL TO THE BRAZILIAN ALUMINUM CAN INDUSTRY

In accordance with the CAPM (Capital Asset Pricing Model), the risk premium (RP) that will be used in this application is given by:

$$\text{PR} = \beta . \text{PR}_{\text{Brasil}} \quad (32)$$

A good representation of risk free rates (*r*) are the sovereign 10 year bonds of the Brazilian government, currently, at around 10.27% per year, according to Bloomberg data. To this value one must discount the value referent to Brazil's default risk, which is represented by the EMBI+ index (*Emerging Markets Bonds Index Plus*), currently standing at 1.87% (this value refers to the risk in Brazil according to site *portalbrasil.net*).

Thus, a good approximation for the risk free rate for Brazil is the difference between those values, that is, *r*=8,25%. The estimated risk premium (RP) for Brazil is 2.63% per year (*Bloomberg*). When calculating the sector's beta, one uses the estimate presented in the report of the Rexam firm (2012), extracted from Bloomberg's database. The value of the beta used in evaluating the firm was 0.9. Thus the annual RP of the aluminum can industry was estimated at 2.37%.

The parameters of the multiplicative stochastic shock, $\tilde{X}_t$, representing uncertainty in the aluminum can industry, must be estimated. Supposing $\tilde{X}_t$ follows a Geometric Brownian Motion (GBM) in the form of equation (1).

The Brazilian Association of Manufacturers of Highly Recyclable Cans (Abralatas) publishes data regarding the sector in Brazil. As the Brazilian can industry is relatively new, there is limited historic data concerning consumption and production. Dixit and Pindyck (1994) recommend long series (two decades or more) to estimate trends. There is information regarding consumption of cans from 1997 to 2013, which is a very short time interval and probably the trend estimates would not be representative of the sector. Instead of calculating trends, a more qualitative approach shall be conducted. Rexam's Annual Report (Rexam, 2012) indicates that between the years of 2014 and 2016 growth in Brazil is estimated to be between five and six percentage points. Since most of this growth is due to international events such as the 2016 Olympic Games, an inferior growth was assumed, equal to three percentage points, to represent long term growth. Therefore: α=3%.

In contrast to the trend calculus, an estimate for volatility does not require a large quantity of data. Even with a limited time interval for the aluminum can sector, the linear regression of the data shall result in a good estimate for the variance of the volatility estimator $var(\hat{\sigma})$, since it is proportional to ($\frac{2\sigma^4}{n}$).

Assuming that consumption in the aluminum can market (Q_T) follows a GBM, the equation in discrete time shall be given by

$$\ln Q_t = a + \ln Q_{t-1} + \varepsilon_t \qquad (33)$$

where ε_t is independent and equally distributed and follows a Normal distribution $N(0,\sigma 2)$.

The estimator of volatility ($\hat{\sigma}$) is equal to:

$$\hat{\sigma}^2 = var\left[ln\left(\frac{Q_t}{Q_{t-1}}\right)\right] \qquad (34)$$

Based on the annual data from this sector from 1990 to 2012, available through the Abralatas website, the square root of data variance is calculated to find an estimate for annual volatility. Therefore: $\hat{\sigma}$ =19,19%.

The demand function is linear and the deterministic parties is given by equation (23), where X.P(Q_T) is the price of an aluminum can, in dollars, in Brazil. To estimate the parameters of the linear function, data was collected from the annual reports of the three firms present in the sector. Considering the total revenue of these firms in Brazil and quantities sold during the years of 2011 and 2012, it was possible to find a line that represents the Brazilian inverse demand for aluminum cans. Tables 2 and 3 represent the total quantities produced in industry and total net earnings respectively.

TABLE 2 – Annual production of aluminum cans in Brazil in billions of units

Year	Q_T (bilhões)
2011	18.50
2012	19.40

TABLE 3 – Annual net earnings of firms in Brazil (US$ million)

Year	Ball	Crown	Rexam
2011	420.38	592.96	1.132,43
2012	432.36	593.22	1.063,58

Upon calculation, the unit price for 2011 was US$ 0.12 and US$ 0.11 in 2012. Therefore, the estimated parameters for the inverse demand function are *a*=0,26 e *b*=0,0079.

This paper assumes the premise of equal investment among the firms for their expansion in production of aluminum cans. Since the domestic firms import machinery and equipment for building new aluminum factories, this is justified as they normally deal with the same suppliers. As per the firms' annual reports, market studies, and sectorial news, the value needed to achieve their investment is quite similar among the competitors. The firms must incur a cost of approximately US$100 million to implement a factory with an annual production capacity of 1 billion aluminum cans. Therefore, the investment in billions of dollars shall be $I = 0.1$.

During the calculation of unit production costs (before expansion), information from the annual reports of each one of the firms was used. The unit operational cost of the firms was estimated by multiplying the unit price of sale by the complement of operational margin of each firm. The average operational margin for each firm shall be used. Based on data from Tables 4 and 5, the calculus of the operational costs (US$/can), assuming the 2012 unit prices (US$ 0.11), we have $c_h^0 = 0{,}0965;\ c_{int}^0 = 0{,}0961\ e\ c_l^0 = 0{,}0942$.

TABLE 4 – Operating margin of firms in the sector

Firm		Revenue (millions)	Operating profit (millions)	Operating margin
Rexam	2012	£ 3.885	£ 456	11,7%
	2011	£ 3.786	£ 447	11,8%
Ball	2012	US$ 6.492	US$ 742	11,4%
	2011	US$6.434	US$ 725	11,3%
Crown	2012	US$4.906	US$665	13,6%
	2011	US$4.803	US$637	13,3%

TABLE 5 – Unit operating cost of firms in the sector

Firm	Operating Margin	Calculations	Operational Unit Cost (c_i^0)
Rexam (*int*)	11,75%	88,25% x 0,11	US$ 0,0961
Ball (*h*)	11,35%	88,65% x 0,11	US$ 0,0965
Crown (*l*)	13,45%	86,55% x 0,11	US$ 0,0942

The expansion investment shall be modeled through a reduction in the firms' operational costs. As they compete in terms of quantity, according to the Cournot model, a reduction in cost will lead to the firm increasing the quantity produced. The decrease in operational costs due to investment shall be considered to be 5%. The operational unit costs after expansion shall be given by $c_h^1 = 0{,}0917;\ c_{int}^1 = 0{,}0913\ e\ c_l^1 = 0{,}0895$.

In the context of Cournot's asymmetric competition, the profit for firm i shall be given by equation (31). In the present case, since n is equal to 3, the profits of the firms for each situation are represented in Table 6. Due to competitive advantage and the assumption $X_0 < \min\{X_L^P; X_L^M\}$, the lowest cost firm shall invest as the Leader, intermediate cost firm as the First Follower, and firm with highest cost as the Second Follower.

TABLE 6 – Profit of firms in the four market situations

Firm	Profit (US$ billions)
No Firm Invested	
Firm	Profit (US$ billions)
$\bar{\pi}^0_{L_l}$	0.234
$\bar{\pi}^0_{F_{int}}$	0.215
$\bar{\pi}^0_{FF_h}$	0.210
Only the Leader Invested	
Firm	Profit (US$ billions)
$\bar{\pi}^1_{L_l}$	0.274
$\bar{\pi}^1_{F_{int}}$	0.202
$\bar{\pi}^1_{FF_h}$	0.198
Leader and First Follower Invested	
Firm	Profit (US$ billions)
$\bar{\pi}^2_{L_l}$	0.260
$\bar{\pi}^2_{F_{int}}$	0.241
$\bar{\pi}^2_{FF_h}$	0.186
All firms invested	
Firma	Profit (US$ billions)
$\bar{\pi}^3_{L_l}$	0.247
$\bar{\pi}^3_{F_{int}}$	0.227
$\bar{\pi}^3_{FF_h}$	0.223

The firm which has the cost advantage shall produce more than the firm which has a higher cost, thus each firm's profits shall be different. After expansion of the low cost firm as the Leader, both its production and profit shall increase. The increase in production shall decrease the unit market price, consequently reducing the deterministic profits of the intermediate cost and high cost firms.

When the intermediate cost firm's trigger is reached, it will then act on its option for expansion becoming the First Follower (*F*). This in turn leads to a reduction in the profits of the low cost firm (which had previously increased its production capacity), as well as the profits of the high cost firm, due to the reduction of the market price. Following the same reasoning, the expansion of the high cost firm, Second Follower (*FF*), shall increase its profit and therefore reduce the other firms' profit.

When analyzing optimal point games, according to Chevalier-Roignant and Trigeorgis (2011), the first step is to find the trigger for the Second Follower firm (*FF*). Previously, we presented the trigger for the Second Follower firm X_{FF}, given by equation (10). Since that firm is the last one to expand its production, it does not need to concern itself with threats of preemption. The assumption of the model, $X_0 < \min\{X_L^P; X_L^M\}$, implies that the firm with the higher cost *h* is the Second Follower. Therefore, the trigger shall be:

$$X_{FF} = \frac{\beta_1}{\beta_1 - 1} \cdot \frac{\delta \cdot I}{\bar{\pi}^3_{FF_h} - \bar{\pi}^2_{FF_h}} = \frac{2{,}47}{2{,}47 - 1} \cdot \frac{7{,}63\% \cdot 0{,}100}{0{,}223 - 0{,}186} = 0{,}348 \tag{35}$$

However, the trigger for the First Follower shall be the minimum one between the trigger calculated in a monopolistic manner and preemption trigger. Therefore, based on equations (13), (14), and (15), the monopolistic trigger for the intermediate cost firm shall be given by equation (36):

$$X_{F_{int}}^{M} = \frac{\beta_1}{\beta_1 - 1} \cdot \frac{\delta \cdot I}{\bar{\pi}_{F_{int}}^{2} - \bar{\pi}_{F_{int}}^{1}} = \frac{2{,}47}{2{,}47 - 1} \cdot \frac{7{,}76\% \cdot 0{,}100}{0{,}241 - 0{,}202} = 0{,}336 \tag{36}$$

To calculate the preemption trigger, one must find the point value where the high cost *h* firm's value as First Follower (F_h) is equal to its value as the Second Follower (FF_h). To reach this equality, one must find the value for the profit flows wherein the low cost firm is the Leader, high cost firm is the First Follower and intermediate cost firm is the Second Follower. Table 7 represents these profits.

TABLE 7 – Profit of the firms considering high-cost Firm as First Follower

No Firm Invested	
Firm	Profit (US$ billions)
$\bar{\pi}_{L_l}^{0}$	0.234
$\bar{\pi}_{F_h}^{0}$	0.210
$\bar{\pi}_{FF_{int}}^{0}$	0.215
Only the Leader Invested	
Firm	Profit (US$ billions)
$\bar{\pi}_{L_l}^{1}$	0.274
$\bar{\pi}_{F_h}^{1}$	0.198
$\bar{\pi}_{FF_{int}}^{1}$	0.202
Leader and First Follower Invested	
Firm	Profit (US$ billions)
$\bar{\pi}_{L_l}^{2}$	0.260
$\bar{\pi}_{F_h}^{2}$	0.236
$\bar{\pi}_{FF_{int}}^{2}$	0.190
All firms Invested	
Firm	Profit (US$ billions)
$\bar{\pi}_{L_l}^{3}$	0.247
$\bar{\pi}_{F_h}^{3}$	0.223
$\bar{\pi}_{FF_{int}}^{3}$	0.227

Calculating the trigger for the intermediate cost firm as the Second Follower ($X_{FF_{int}}$), we obtain that:

$$X_{FF_{int}} = \frac{\beta_1}{\beta_1 - 1} \cdot \frac{\delta \cdot I}{\bar{\pi}_{FF_{int}}^{3} - \bar{\pi}_{FF_{int}}^{2}} = \frac{2{,}47}{2{,}47 - 1} \cdot \frac{7{,}62\% \cdot 0{,}100}{0{,}227 - 0{,}190} = 0{,}346. \tag{37}$$

Equaling the values of the high cost firm as the First Follower (F_h) and Second Follower (FF_h^0), we get the following equation (38):

$$\left(\frac{1}{X_{F_{int}}}\right)^{\beta_1}\cdot\frac{X_{F_{int}}\left[\bar{\pi}^2_{FF_h}-\bar{\pi}^1_{FF_h}\right]}{\delta}+\left(\frac{1}{X_{FF_h}}\right)^{\beta_1}\cdot\left[\frac{X_{FF_h}\left[\bar{\pi}^3_{FF_h}-\bar{\pi}^2_{FF_h}\right]}{\delta}-I\right] = \left(\frac{1}{X_{F_h}}\right)^{\beta_1}\cdot\left[\frac{X_{F_h}\left[\bar{\pi}^2_{F_h}-\bar{\pi}^1_{F_h}\right]}{\delta}-I\right]+\left(\frac{1}{X_{FF_{int}}}\right)^{\beta_1}\cdot\frac{X_{FF_{int}}\left[\bar{\pi}^3_{F_h}-\bar{\pi}^2_{F_h}\right]}{\delta} \tag{38}$$

Replacing the values in the equation and using the *Solver* function in Excel, we find the trigger $X_{F_{int}}$, that is, the preemption trigger of the intermediate cost firm, $X^P_{F_{int}} = 0{,}298$.

The intermediate cost firm shall invest in:

$$X_{F_{int}} = \min\{X^M_{F_{int}}; X^P_{F_{int}}\} = \min\{0{,}336; 0{,}298\} = 0{,}298\,. \tag{39}$$

The threat of preemption implies that the intermediate cost firm cannot wait for the point which maximizes its value as the First Follower. Since its cost advantage is not large enough, it needs to invest before in order not to be anticipated by the high cost firm.

The trigger for the Leader firm shall be the minimum between the trigger calculated in a monopolistic manner and the preemption trigger. The monopolistic trigger of the low cost firm is given by:

$$X^M_{L_l} = \frac{\beta_1}{\beta_1 - 1}\cdot\frac{\delta\cdot I}{\bar{\pi}^1_{L_l}-\bar{\pi}^0_{L_l}} = \frac{2{,}47}{2{,}47-1}\cdot\frac{7{,}76\%\cdot 0{,}100}{0{,}274-0{,}234} = 0{,}320\,. \tag{40}$$

To calculate the preemption trigger, one must find the value for X_t at which intermediate cost the firm is indifferent between investing as the Leader or waiting to invest in $X_{F_{int}}$, that is, the point at which the value of the intermediate cost firm as the Leader (L_{int}) is equal to its value as the First Follower (F_{int}). To find the value for $X_{L_{int}}$ that solves that equation, one must find the trigger of the low cost firm acting as the First Follower (X_{F_l}). It is also necessary to find the value for the profit flows in this case, wherein the intermediate cost firm is the Leader, low cost firm is the First Follower, and high cost firm is the Second Follower. Table 8 summarizes the profits.

TABLE 8 – Profit of firms considering intermediate-cost firm as First Follower

No Firm Invested	
Firm	Profit (billions)
$\bar{\pi}^0_{L_{int}}$	0.215
$\bar{\pi}^0_{F_l}$	0.234
$\bar{\pi}^0_{FF_h}$	0.210
Only Leader Invested	
Firm	Profit (billions)
$\bar{\pi}^1_{L_{int}}$	0.254
$\bar{\pi}^1_{F_l}$	0.221
$\bar{\pi}^1_{FF_h}$	0.198
Leader and First Follower Invested	
Firm	Profit (billions)
$\bar{\pi}^2_{L_{int}}$	0.241
$\bar{\pi}^2_{F_l}$	0.260
$\bar{\pi}^2_{FF_h}$	0.186
All firms Invested	
Firm	Profit(billions)
$\bar{\pi}^3_{L_{int}}$	0.227
$\bar{\pi}^3_{F_l}$	0.247
$\bar{\pi}^3_{FF_h}$	0.223

Now, it is possible to calculate the trigger for the low cost firm acting as the First Follower (X_{F_l})

$$X_{F_l} = \frac{\beta_1}{\beta_1 - 1} \cdot \frac{\delta \cdot I}{\bar{\pi}^2_{F_l} - \bar{\pi}^1_{F_l}} = \frac{2{,}47}{2{,}47 - 1} \cdot \frac{7{,}62\% \cdot 0{,}100}{0{,}260 - 0{,}221} = 0{,}328. \tag{41}$$

Next, we may equalize the values of the intermediate cost firm investing as the Leader and First Follower, that is:

$$L^0_{int}(X_T) = F^0_{int}(X_T) \tag{42}$$

$$\begin{aligned}\left(\frac{1}{X_{L_{int}}}\right)^{\beta_1} \cdot \left[\frac{X_{L_{int}}\left[\bar{\pi}^1_{L_{int}} - \bar{\pi}^0_{L_{int}}\right]}{\delta} - I\right] + \left(\frac{1}{X_{F_l}}\right)^{\beta_1} \cdot \frac{X_{F_l}\left[\bar{\pi}^2_{L_{int}} - \bar{\pi}^1_{L_{int}}\right]}{\delta} \\ = \left(\frac{1}{X_{L_l}}\right)^{\beta_1} \cdot \frac{X_{L_l}\left[\bar{\pi}^1_{F_{int}} - \bar{\pi}^0_{F_{int}}\right]}{\delta} + \left(\frac{1}{X_{F_{int}}}\right)^{\beta_1} \cdot \left[\frac{X_{F_{int}}\left[\bar{\pi}^2_{F_{int}} - \bar{\pi}^1_{F_{int}}\right]}{\delta} - I\right]\end{aligned} \tag{43}$$

Replacing all the values calculated in the equation and using the *Solver* function in *Excel*, one obtains the trigger $X_{L_{int}}$, that is, the preemption trigger for the low cost firm $X^P_{L_l}$, thus:

The low cost firm shall invest in:

$$X_{L_l} = \min\{X^M_{L_l}; X^P_{L_l}\} = \min\{0{,}320; 0{,}290\} = 0{,}290. \tag{44}$$

The threat of preemption implies that the low cost firm cannot wait for the point which maximizes its value as the Leader. Since its cost advantage is not big enough, it needs to invest before in order not to be anticipated by the intermediate cost firm.

At time t = 0, all three firms are active in the market. When the stochastic shock X_T reaches the value of 0.290, the low cost firm shall invest the amount *I* and expand its production. As long as the stochastic shock is greater than 0.290 and lesser than 0.298, only the low cost firm will benefit from increasing production. Only when the shock reaches 0.298, the intermediate cost firm exercise its option to increase production. As for the high cost firm, it will only invest when the shock reaches 0.348. Figure 1 illustrates the triggers wherein the Leader firm with a low cost, incapable of waiting for the point at which it would maximize its value, shall invest during its preemption point $T^P_{L_l}$ referent to the monopolistic trigger $T^P_{L_l}$. As for the First Follower firm of intermediate cost, since it cannot wait to invest at the point in which it would maximize its value either, it shall invest during its preemption time point $T^P_{F_{int}}$ referent to the monopolistic trigger point $T^M_{F_{int}}$. Finally, due to lack of preemption threat, the Second Follower firm of high cost shall invest at its monopolistic time point $T^M_{F_{int}}$.

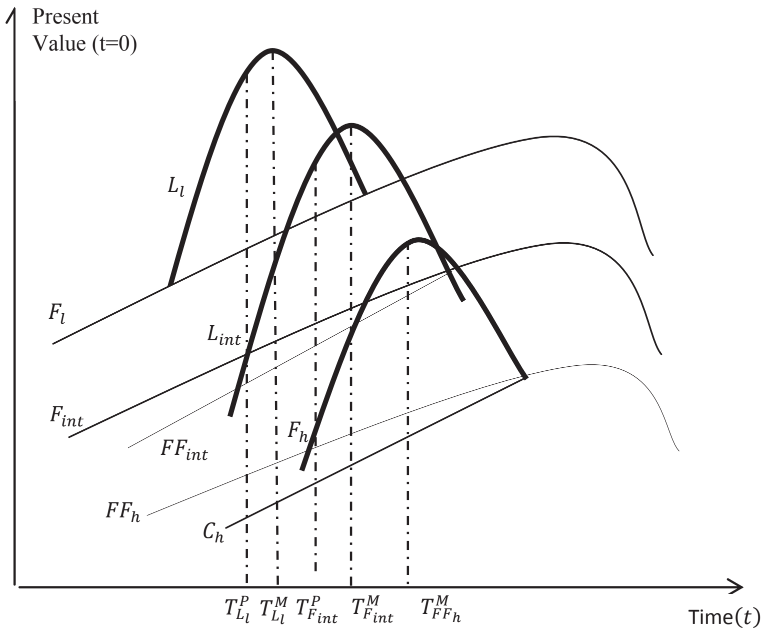

FIGURE 1 – Preemption and monopolistic triggers of firms

5.1 Sensitivity analysis

Sensitivity analysis is used to study the behavior of the results of a study the extent to which the model assumptions vary. This analysis makes it possible to evaluate the degree of confidence in the results when there is uncertainty in the assumptions of the data and results. A comparison of the variation in monopolistic triggers and preemption of the three firms regarding the variables estimated in this section. The triggers are used as reference:

TABLE 9 – The Triggers

	X_i^P	X_i^M
L_l	0.290	0.320
F_{int}	0.298	0.336
FF_h	–	0.348

For each case, Tables 10, 11, 12 and 13 show the amounts and percentage changes of monopoly triggers and preemption of low-cost, intermediate cost and high cost firms, for the risk-free rate, to the tendency for volatility, and the demand function, respectively.

TABLE 10 – Sensitivity analysis of risk-free rate

	$r_{5\%} = 8.66\%$		$r_{10\%} = 9.08\%$		$r_{15\%} = 9.49\%$		$r_{20\%} = 9.90\%$	
	X_i^P	X_i^M	X_i^P	X_i^M	X_i^P	X_i^M	X_i^P	X_i^M
L_l	0.302	0.332	0.313	0.345	0.325	0.357	0.336	0.370
Δ	3.99%	3.97%	7.96%	7.92%	11.9%	11.8%	15.9%	15.8%
F_{int}	0.310	0.349	0.322	0.362	0.334	0.376	0.345	0.389
Δ	4.00%	3.97%	7.98%	7.92%	11.9%	11.8%	15.9%	15.8%
FF_h	–	0.362	–	0.375	–	0.389	-	0.403
Δ		3.97%		7.92%		11.8%		15.8%

An increase of 20% in the risk-free rate causes the trigger to be changed, positively, around 16%. An increase in the risk-free rate will be postponing the optimal timing of business investment, i.e. investments will be justified only in a more lucrative market than previously.

TABLE 11 – Sensitivity analysis of the trend

	$\alpha_{5\%} = 3.15\%$		$\alpha_{10\%} = 3.30\%$		$\alpha_{15\%} = 3.45\%$		$\alpha_{20\%} = 3.60\%$	
	X_i^P	X_i^M	X_i^P	X_i^M	X_i^P	X_i^M	X_i^P	X_i^M
L_l	0.288	0.317	0.286	0.315	0.284	0.313	0.282	0.311
Δ	-0.689%	-0.670%	-1.37%	-1.33%	-2.03%	-1.98%	-2.69%	-2.62%
F_{int}	0.296	0.334	0.294	0.331	0.292	0.329	0.290	0.327
Δ	-0.696%	-0.670%	-1.38%	-1.33%	-2.05%	-1.98%	-2.71%	-2.62%
FF_h	–	0.346	–	0.343	–	0.341	-	0.339
Δ		-0.670%		-1.33%		-1.98%		-2.62%

An increase of 20% in the trend of geometric brownian motion causes the trigger to be changed negatively around 2.6%. The higher the trend of GBM, faster the market will grow and become more profitable. The result is poorly sensitive to this variable.

TABLE 12 – Sensitivity Analysis of Volatility

	$\sigma_{5\%} = 20.15\%$		$\sigma_{10\%} = 21.11\%$		$\sigma_{15\%} = 22.07\%$		$\sigma_{20\%} = 23.03\%$	
	X_i^P	X_i^M	X_i^P	X_i^M	X_i^P	X_i^M	X_i^P	X_i^M
L_l	0.297	0.327	0.304	0.335	0.311	0.343	0.318	0.351
Δ	2.33%	2.36%	4.70%	4.77%	7.13%	7.23%	9.61%	9.73%
F_{int}	0.305	0.344	0.312	0.352	0.319	0.360	0.326	0.368
Δ	2.31%	2.36%	4.68%	4.77%	7.09%	7.23%	9.56%	9.73%
FF_h	–	0.356	–	0.364	–	0.373	-	0.382
Δ		2.36%		4.77%		7.23%		9.73%

An increase of 20% in the volatility means positive changes in the trigger, around 10%. The volatility negatively impacts the value of the variable β_1. On the other hand, a negative variation in β_1 value increases the value of the trigger. Thus, the volatility impacts the trigger value in the same direction.

TABLE 13 – Sensitivity Analysis of the Demand Function

	$a_{5\%} = 0.275$		$a_{10\%} = 0.288$		$a_{15\%} = 0.302$		$a_{20\%} = 0.315$	
	X_i^P	X_i^M	X_i^P	X_i^M	X_i^P	X_i^M	X_i^P	X_i^M
L_l	0.270	0.298	0.238	0.279	0.238	0.262	0.225	0.247
Δ	-6.76%	-6.82%	-17.8%	-12.8%	-17.8%	-18.0%	-22.4%	-22.6%
F_{int}	0.277	0.311	0.243	0.290	0.243	0.272	0.230	0.256
Δ	-6.96%	-7.27%	-18.3%	-13.6%	-18.3%	-19.0%	-23.0%	-23.9%
FF_h	–	0.322	–	0.299	–	0.279	-	0.262
Δ		-7.54%		-14.0%		-19.7%		-24.6%

An increase of 20% in variable of demand function means negative changes in the trigger, around 22.6%. The profit is positively affected by the higher price. In this scenario, firms will invest in an early date, and therefore the trigger is negatively impact. This is the more sensitive variable of the model.

6 CONCLUSIONS

This study examined how the threat of preemption in a context of oligopolistic competition may interfere with the optimal decision point for investment. The model was applied to the aluminum can industry, wherein there are three main active firms. It was possible to compare the impact of preemption in the firms' trigger.

The accomplishment of this study helps to emphasize the importance, in a dynamic model of obtaining a competitive advantage in costs. As for the static case, the firm with the lowest cost is able to absorb a greater market profit, as it possesses a superior margin. In the dynamic case, monopolistic profits are possibly obtained, or advantages from an isolated expansion may be enjoyed during a greater time period. In case of a very large competitive advantage, the rival's threat of preemption may be considered irrelevant and the firm shall invest in the monopolistic period ignoring the possibility of a rival entering the market. When the cost advantage is not large enough for the firms to invest at their monopolistic trigger point, the firm needs to enter a preemption point earlier , due to strategic interaction and market rivalry.

When applying the model, we assumed an investment equal to US$ 100 million for the three firms in the market. The firms' production was modeled through the Cournot equilibrium. The marginal production cost was estimated by information published by the firms in the market. The result was a relatively similar output for each one of the three firms in the Brazilian sector.

In Brazil, there is a great difference between the market shares of the firms. Whereas in the studied model, all the firms possess a market share close to 30% in the Brazilian market, the difference is significant, wherein each of their shares is approximately 20%, 25%, and 55%. It is interesting to note that it is not the firm with the lowest cost that sells more in the market; however, greater market share is justified by historical reasons and also due to long term contracts between the firms and biggest buyers of aluminum cans.

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